Sato, R.
Osaka J. Math.
36 (1999), 485-495

ELEMENTARY PROOFS OF POINTWISE ERGODIC THEOREMS FOR MEASURE PRESERVING TRANSFORMATIONS

RYOTARO SATO

(Received August 04, 1997)

1. Introduction

Recently Kamae and Keane [5] have obtained a simple proof of Hopf's ratio ergodic theorem using an idea due to Kamae [4] and Shields [9]. In this paper the same idea will be applied to obtain elementary proofs of pointwise ergodic theorems for superadditive processes relative to measure preserving transformations. The main tool is a maximal ergodic theorem for superadditive processes, whose proof has been motivated by Jones [3] and Akcoglu and Krengel [1]. For related results we refer the reader to [2], [6] and [10] (see also [7] and §1.5 of [8]).

2. A maximal ergodic theorem

Let $(X, \mathcal{F}, \mu)$ be a σ-finite measure space and $T : X \to X$ be a measure preserving transformation. As usual, two measurable functions f and g are not distinguished provided that $f(x) = g(x)$ a.e. on X. A family $\mathbf{F} = \{F_{i,k} : 0 \le i < k\}$ of measurable functions is called a *superadditive process* in $L_1(\mu)$ if it satisfies

(i) $F_{i,k} \circ T = F_{i+1,k+1}$ for $0 \le i < k$,

(ii) $F_{i,l} \ge F_{i,k} + F_{k,l}$ for $0 \le i < k < l$,

(iii) the functions $F_{i,k}$ are all integrable and $\gamma(\mathbf{F}) = \sup\{n^{-1} \int_X F_{0,n}\, d\mu : n \ge 1\} < \infty$.

It should be noted here that since $\int_X F_{0,n+m}\, d\mu \ge \int_X F_{0,n}\, d\mu + \int_X F_{0,m}\, d\mu$ by (i) and (ii), it follows easily that $\gamma(\mathbf{F}) = \lim_n n^{-1} \int_X F_{0,n} d\mu$. (This is standard. See e.g. Lemma 1.5.1 of [8].) $\mathbf{F}$ is called a *subadditive process* in $L_1(\mu)$, if $-\mathbf{F} = \{-F_{i,k}\}$ is a superadditive process in $L_1(\mu)$, and an *additive process* in $L_1(\mu)$ if $\mathbf{F}$ is a both superadditive and subadditive process in $L_1(\mu)$. $\mathbf{F}$ is called an *extended superadditive process* if it satisfies (i) and (ii), but not necessarily (iii).

The following maximal ergodic theorem is basic throughout this paper.

Theorem 1. Let $\mathbf{F} = \{F_{i,k} : 0 \le i < k\}$ be a nonnegative superadditive process in $L_1(\mu)$. If g is a nonnegative measurable function on X and