ON GALOIS EXTENSION OF RINGS

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To the memory of TADASI NAKAYAMA

1. Introduction. Let Λ be a ring and G a finite group of ring automorphisms of Λ. The totality of elements of Λ which are left invariant by G is a subring of Λ. We call it the G-fixed subring of Λ. Let $\Delta = \Delta(\Lambda, G) = \sum_{\sigma \in G} \oplus \Lambda u_\sigma$ be the crossed product of Λ and G with trivial factor set, i.e. $\{u_\sigma\}$ is a Λ-free basis of Δ and $u_\sigma u_\tau = u_{\sigma\tau}$, $u_\sigma \lambda = \sigma(\lambda) u_\sigma$ for $\lambda \in \Lambda$, and let Γ be a subring of the G-fixed subring of Λ which has the same identity as Λ. Then we have a ring homomorphism

$$\delta: \ \Delta(\Lambda, G) \to \mathrm{Hom}_\Gamma^r(\Lambda, \Lambda)$$

defined by $\delta(\lambda u_\sigma)(x) = \lambda\sigma(x)$, where $\mathrm{Hom}_\Gamma^r(\Lambda, \Lambda)$ is the Γ-endomorphism ring of Λ regarded as Γ-right module.

In [4], we generalized the notion of Galois extension, which was first defined by Auslander and Goldman [1] for commutative rings, to non commutative case, and discussed the Galois theory for non commutative rings. Our definition of Galois extension is as follows. A ring Λ is called a *Galois extension* of Γ relative to G if the following conditions are satisfied:

I. Γ is the G-fixed subring of Λ,

II. Λ is a finitely generated projective Γ-right module,

III. δ is an isomorphism of $\Delta(\Lambda, G)$ to $\mathrm{Hom}_\Gamma^r(\Lambda, \Lambda)$.

On the other hand, Chase, Harrison and Rosenberg [3] gave another definition of Galois extension, which is equivalent to the above in commutative case, and developed a Galois theory for commutative rings. In order to state other definition, we set $Tr(x) = \sum_{\sigma \in G} \sigma(x)$ for $x \in \Lambda$. Then Λ is called to be a Galois extension of Γ relative to G if the following two conditions are satisfied:

CHR I. $\Gamma = Tr(\Lambda)$.

CHR II. There exist $x_1, x_2, \ldots$ and x_r and $y_1, y_2, \ldots y_r$ in Λ such that

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