ANALYTIC FUNCTIONS ON SOME RIEMANN SURFACES, II

KIKUJI MATSUMOTO

To Professor Kinjiro Kunugi on the occasion of his 60th birthday

1. In their paper [12], Toda and the author have concerned themselves in the following

THEOREM OF KURAMOCHI. *Let R be a hyperbolic Riemann surface of the class O_{HB}(O_{HD}, resp.). Then, for any compact subset K of R such that $R-K$ is connected, $R-K$ as an open Riemann surface belongs to the class O_{AB}(O_{AD}, resp.)* (Kuramochi [4]).

They have raised there the question as to whether there exists a hyperbolic Riemann surface, which has no Martin or Royden boundary point with positive harmonic measure and has yet the same property as stated in Theorem of Kuramochi, and given a positive answer to the Martin part of this question.

The main purpose of this paper is to show that the Royden part is also answered in the positive. In the sequel, we shall investigate covering properties of analytic functions on Riemann surfaces of the class O°_{AD}, which was introduced by Kuroda in his paper [6], give an extension of the D part of Theorem of Kuramochi and, using this extension, give an example of a Riemann surface which answers the Royden part of the question in the positive.

2. Let R be a Riemann surface and let G be a domain on R with smooth relative boundary ∂G clustering nowhere in R. For simplicity, we shall call such a domain G a subregion of R. If G admits no non-constant single-valued analytic function with a finite Dirichlet integral and with real part vanishing continuously on its relative boundary ∂G, we say that G belongs to the class SO_{AD}.

Let $w=f(p)$ be a non-constant single-valued analytic function in a subregion G with relative boundary ∂G of a Riemann surface R. We suppose that

Received May 10, 1963.