CROSSED PRODUCTS AND HEREDITARY ORDERS

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Introduction. Let S be the integral closure of a discrete rank one valuation ring R in a finite Galois extension of the quotient field of R, and denote the Galois group of the quotient field extension by G. It has been proved by Auslander and Rim in [4] that the trivial crossed product $\Delta(1, S, G)$ is an hereditary order for tamely ramified extensions S of R, and that $\Delta(1, S, G)$ is a maximal order if and only if S is an unramified extension of R. The purpose of this paper is to study the crossed product $\Delta(f, S, G)$ where $[f]$ is any element of $H^2(G, U(S))$ and S is a tamely ramified extension of R with multiplicative group of units $U(S)$.

The main theorem of Section 1 states that for an extension S of R the following three properties are equivalent:

(1) S is a tamely ramified extension of R

(2) the crossed product $\Delta(f, S, G)$ is an hereditary order for each $[f]$ in $H^2(G, U(S))$

(3) the trivial crossed product $\Delta(1, S, G)$ is an hereditary order.

We then give an example to show that not every hereditary order is equivalent to a crossed product over a tamely ramified extension.

In Section 2 we study the number of maximal two-sided ideals in the crossed product $\Delta(f, S, G)$. It has been proved by Harada in [6] that the number of maximal two-sided ideals in an hereditary order Λ over a discrete rank one valuation ring R in a central simple algebra Σ over the quotient field of R is equal to the length of a saturated chain of orders over R in Σ containing Λ. This is the main motivation for our study. Given a crossed product $\Delta(f, S, G)$ over a tamely ramified extension S of R we define the conductor group H_f of $\Delta(f, S, G)$ to be a certain subgroup of the inertia group of a maximal ideal of S. Then we show that the number of maximal two-sided ideals in $\Delta(f, S, G)$ is equal to the order of the conductor group H_f. In particular, the number

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