

ANALYTIC FUNCTIONS ON SOME RIEMANN SURFACES

NOBUSHIGE TODA and KIKUJI MATSUMOTO

1. Some years ago, Kuramochi gave in his paper [5] a very interesting theorem, which can be stated as follows.

THEOREM OF KURAMOCHI. *Let R be a hyperbolic Riemann surface of the class $O_{HR}(O_{HD}, \text{resp.})$. Then, for any compact subset K of R such that $R - K$ is connected, $R - K$ as an open Riemann surface belongs to the class $O_{AB}(O_{AD}, \text{resp.})$.*

The excellent work of Constantinescu and Cornea [2] clarified that as for the B part of this theorem, the existence of a *bounded minimal harmonic function*, that is, the existence of a Martin boundary point with positive harmonic measure and, as for the D part, the existence of an *HD -minimal function* are essential. Later Kusunoki and Mori [8] and Nakai [9] proved the equivalence of the existence of an *HD -minimal function* and the existence of a point with positive harmonic measure in Royden's harmonic boundary of the Riemann surface R . But there remains the question as to whether there exists a hyperbolic Riemann surface which has no Martin or Royden boundary point with positive harmonic measure and has yet the same property as stated in Theorem of Kuramochi.

The main purpose of this paper is to give a positive answer to the Martin part of the above question. In the sequel, we shall give an extension of the B part of Theorem of Kuramochi and, using this extension, construct an open Riemann surface, as a covering surface of the complex plane, which answers the Martin part of the question in the positive.

2. First we shall be concerned with the boundary behaviour of bounded analytic functions on a Riemann surface of the class O_{AB}^o . The class O_{AB}^λ was introduced and investigated first by Kuroda in his paper [7]. It is a class of