A CRITERION FOR A SET AND ITS IMAGE UNDER QUASICONFORMAL MAPPING TO BE OF α ($0<\alpha\leqq 2$)-DIMENSIONAL MEASURE ZERO

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Let $w=w(z)$ be any K-quasiconformal mapping (in the sense of Pfluger-Ahlfors) of a domain D in the z-plane into the w-plane. Since $w=w(z)$ is a measurable mapping (vid. Bers [1]), it transforms any set of Hausdorff's 2-dimensional measure zero in D into such another one. However, A. Mori [5] showed that for $0<\alpha\leqq 2$, any set of $\frac{\alpha}{K}$-dimensional measure zero in a Jordan domain D is transformed by $w=w(z)$ into a set of α-dimensional measure zero. Further, Beurling and Ahlfors [2] proved that even the set of 1-dimensional measure zero on a segment S in D is not always transformed into such another one under $w=w(z)$ transforming S into another segment.

In this paper motivated by the above results, by extending our argument in the previous paper [3] where the following lemma due to Teichmüller is very useful, we shall give a criterion for both some closed set E in a Jordan domain D and its image set by any K-quasiconformal mapping $w=w(z)$ of D to be of α-dimensional measure zero, where $0<\alpha\leqq 2$.

Lemma (Teichmüller [6]). If one of the complementary continua of a doubly connected domain R contains $z=0$ and $z=re^{i\theta}$ and the other contains $z=\infty$ and $z=\rho e^{i\varphi}$, then it holds

$$\operatorname{mod} R\leqq \log \Psi\left(\frac{\rho}{r}\right),$$

where $\log\Psi(P)$ means the modulus of Teichmüller's extremal domain.

1. Let E be a compact set in the complex plane and let its complementary set be a connected domain G.

A set $\{R_n^{(j)}\}$ ($j=1,2,\ldots,\nu(n)<\infty$; $n=1,2,\ldots$) of doubly connected domains $R_n^{(j)}$ will be simply referred a system inducing an exhaustion of G if it satisfies the following conditions:

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