THE SPACE OF DIRICHLET-FINITE SOLUTIONS OF THE EQUATION $\Delta u = Pu$ ON A RIEMANN SURFACE

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Introduction and preliminaries

1. Let R be an open Riemann surface. By a *density* P on R we mean a non-negative and continuously differentiable functions $P(z)$ of local parameters $z = x + iy$ such that the expression $P(z)dxdy$ is invariant under the change of local parameters z. In this paper we always assume that $P \not\equiv 0$ unless the contrary is explicitly mentioned. We consider an elliptic partial differential equation

$$(1) \qquad \Delta u = Pu, \qquad \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2,$$

which is invariantly defined on R. For absolutely continuous functions f in the sense of Tonelli defined on R, we denote *Dirichlet integrals* and *energy integrals* of f taken over R by

$$D_R[f] = \iint_R (|\partial f/\partial x|^2 + |\partial f/\partial y|^2)dx\,dy$$

and

$$E_R[f] = \iint_R (|\partial f/\partial x|^2 + |\partial f/\partial y|^2 + P|f|^2)dx\,dy$$

respectively. By a solution of (1) on R we mean a twice continuously differentiable function which satisfies the relation (1) on R. We denote by PB (or PD or PE) the totality of bounded (or Dirichlet-finite or energy-finite) solutions of (1) on R. We also denote by $PBD = PB \cap PD$ and $PBE = PB \cap PE$. If the class X contains no non-constant function, then we denote the fact by $R \in O_X$, where X stands for one of classes PB, PD, PE, PBD or PBE. Here we remark that a constant solution of (1) is necessarily zero, since we have assumed that $P \not\equiv 0$ on R. We also use the notation $R \in O_G$ to denote the fact that R is a

Received October 31, 1960.