ON MULTIPLE TRANSITIVITY OF PERMUTATION GROUPS

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It is well known that a doubly transitive group $\mathfrak{G}$ has an irreducible character χ_1 such that $\chi_1(R) = \alpha(R) - 1$ for any element R of $\mathfrak{G}$ and a quadruply transitive group has irreducible characters χ_2 and χ_3 such that $\chi_2(R) = \frac{1}{2}\alpha(R)(\alpha(R)-3)+\beta(R)$ and $\chi_3(R) = \frac{1}{2}(\alpha(R)-1)(\alpha(R)-2)-\beta(R)$ where $\alpha(R)$ and $\beta(R)$ are respectively the numbers of one cycles and two cycles contained in R. G. Frobenius was led to this fact in the connection with characters of the symmetric groups and he proved the following interesting theorem[1]: if a permutation group $\mathfrak{G}$ of degree n is t-ply transitive, then any irreducible character of the symmetric group of degree n with dimension at most equal to $\frac{t}{2}$ is an irreducible character of $\mathfrak{G}$.

In this paper, we shall prove some theorems of a similar type to the above theorem by G. Frobenius which assert multiple transitivity of permutation groups in connection with characters of the symmetric groups. In § 1 we shall sketch some results on the characters of the symmetric groups by G. Frobenius and I. Schur needed in this paper, and in § 2 we prove our theorems. Further, we treat some special cases in § 3.

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We use following terminologies and notations. $\mathfrak{S}^n$ designates the symmetric groups of degree n (on n letters $\underline{1}, \underline{2}, \ldots, \underline{n}$). $\mathfrak{S}_1^n$ is the subgroup of $\mathfrak{S}^n$ fixing suitable one letter, say $\underline{1}$, and frequently we identify this group with $\mathfrak{S}^{n-1}$. $\alpha_1, \alpha_2, \ldots, \alpha_n$ are rational integer valued class functions of $\mathfrak{S}^n$ such that, for an element R of $\mathfrak{S}^n$, $\alpha_i(R)$ is the number of cycles of length i contained in R, and we say that the type of the element R is $(1)^{\alpha_1(R)}(2)^{\alpha_2(R)} \cdots (n)^{\alpha_n(R)}$. Characters are always ordinary characters and irreducible characters are abso-

1) See [4], § 3.

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