ON THE DIMENSION OF MODULES AND ALGEBRAS, VI

COMPARISON OF GLOBAL AND ALGEBRA DIMENSION

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Throughout this paper all rings are assumed to have unit elements. A ring Λ is said to be semi-primary if its Jacobson radical N is nilpotent and $\Gamma = \Lambda/N$ satisfies the minimum condition. The main objective of this paper is

THEOREM I. *Let Λ be a semi-primary algebra over a field K. Let N be the radical of Λ and $\Gamma = \Lambda/N$. If*

$$\dim \Lambda < \infty \text{ and } (\Gamma : K) < \infty,$$

Then

$$\dim \Lambda = \text{gl.dim}\, \Lambda.$$

Here $\dim \Lambda$ denotes the dimension of Λ as a K-algebra, i.e. $\dim \Lambda = \text{l.dim}_{\Lambda^e} \Lambda$ where $\Lambda^e = \Lambda \otimes_K \Lambda^*$.

We do not know whether the condition $(\Gamma : K) < \infty$ follows from the condition that Λ is a semi-primary ring such that $\text{gl.dim}\, \Lambda = \dim \Lambda < \infty$. The theorem has been previously proven in [3] and [4] under the stronger assumption $(\Lambda : K) < \infty$. In this case it was further shown that Γ is separable (i.e. $\dim \Gamma = 0$). We do not know whether this is true without the assumption $(\Lambda : K) < \infty$.

1. Tensor product of semi-simple algebras

A semi-primary ring Λ with radical N is called *primary* if Λ/N is a simple ring.

PROPOSITION 1. *Let Λ and Σ be rings and $\varphi : \Lambda \longrightarrow \Sigma$ a ring epimorphism. If Λ is a semi-primary ring with radical N, then Σ is a semi-primary ring with radical $\varphi(N)$.*

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