ON SEGREGATED RINGS AND ALGEBRAS

J. P. JANS

Segregated algebras have been nicely characterized by M. Ikeda [4]. In this paper § 1, we consider segregated rings and study the structure of such rings in Theorems 1.1 and 1.2. In § 2, we specialize to the case of segregated algebras of finite dimension over a field. Theorem 2.1 gives a new characterization of such algebras. Theorem 2.2 shows an interesting property of segregated algebras; two segregated algebras S and T, with radicals N and P respectively, are isomorphic if and only if S/N^2 and T/P^2 are isomorphic.

§ 1. Segregated Rings

Following [3], if $\theta: A \to B$ is a ring homomorphism of A onto B, we shall say that B is segregated in A if there exists a subring A' of A such that θ restricted to A' is an isomorphism. Clearly $A = A' + \text{kernel } \theta$ is a direct sum. If B is a ring and M an abelian group we say that M is a (B, B) module if M is both a left and right B module and the associativity condition $\beta(m\gamma) = (\beta m)\gamma$ holds for m in M and β, γ in B.

Consider a class $\mathfrak{C}$ of rings with the property that if B belongs to $\mathfrak{C}$ and A is a subring of B then A belongs to $\mathfrak{C}$.

Definition 1.1: A ring B is *segregated in* $\mathfrak{C}$ will mean B belongs to $\mathfrak{C}$ and, when B is the homomorphic image of D belonging to $\mathfrak{C}$, B is segregated in D.

Definition 1.2: A ring B is *separated in* $\mathfrak{C}$ if, 1. B is segregated in $\mathfrak{C}$, and 2. if B is a subring of D belonging to $\mathfrak{C}$ and M is a subset of D which is a (B, B) module then M is completely reducible as a (B, B) module.

One sees that a separated ring with identity is the direct sum of a finite number of simple ideals with identity and is therefore semisimple. (We would like to say that B is separable if every (B, B) module is completely reducible, and then prove a separable ring is segregated. Since we do not know that this

Received January 9, 1956.