COHOMOLOGY RELATIONS IN SPACES WITH A TOPOLOGICAL TRANSFORMATION GROUP[1]

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1. Introduction

Let Q be a topological transformation group operating on the left of a topological space X. Let us denote by B the orbit space and $p: X \to B$ the projection. p is a continuous and open map of X onto B. For an arbitrary abelian coefficient group G, the continuous map p induces homomorphisms

$$p^*: H^n(B, G) \to H^n(X, G), \quad (n \geqq 0),$$

of the Alexander-Wallace cohomology groups [1][2]. These induced homomorphisms are, in general, not onto isomorphisms. They depend on the manner in which the topological transformation group Q operates on X.

To measure the deviation of these induced homomorphisms p^* from the onto isomorphisms, we introduce, in the present paper, the *weakly residual cohomology groups*

$$H^n_w(X, G), \quad (n \geqq 0).$$

They are invariants depending on X, Q, G and the operations of Q on X. By means of these groups, we shall establish an exact sequence

$$H^0(B, G) \xrightarrow{p^*} \cdots \to H^n(B, G) \xrightarrow{p^*} H^n(X, G) \to H^n_w(X, G) \to H^{n+1}(B, G) \xrightarrow{p^*} \cdots.$$

This indicates that the weakly residual cohomology groups $H^n_w(X, G)$ might play an important role in the further studies of the cohomology structures of the orbit space.

For each point $x \in X$, there is a canonical homomorphism

$$k^*_x: H^n_w(X, G) \to H^n(Q, G), \quad (n \geqq 0).$$

It is proved that if Q is compact and if x and y are two points contained in a compact connected subset of X then $k^*_x = k^*_y$.

2. Preliminaries

Throughout the present paper, let Q be a topological group acting as a

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[2] Numbers in square brackets refer to the bibliography at the end of the paper.