RIEMANN SPACE WITH TWO-PARAMETRIC HOMOGENEOUS HOLONOMY GROUP

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The rotational part of the holonomy group of a Riemann space is called its homogeneous holonomy group. A Riemann space, whose homogeneous holonomy group is one-parametric, was investigated by Liber and an alternative treatment of the same problem was given by S. Sasaki [1]. I will treat here a Riemann space with two-parametric homogeneous holonomy group and prove the following theorem by the method analogous to that of [1]. I thank Prof. T. Ootuki for his kind advice.

THEOREM. *If the homogeneous holonomy group of an n-dimensional Riemann space is two-parametric, the space is a direct sum of two non-flat two-dimensional Riemann spaces and a $n-4$-dimensional flat space; namely the line-element of our space is given by*

$$ds^2 = d\sigma_1^2 + d\sigma_2^2 + \sum_{i=5}^{n}(dx^i)^2$$

where

$$d\sigma_1^2 = \sum_{i,\,j=1,\,2} g_{ij}(x^1,\, x^2)dx^i dx^j, \quad d\sigma_2^2 = \sum_{i,\,j=3,\,4} g_{ij}(x^3,\, x^4)dx^i dx^j.$$

0. Let the line element of a general Riemann space be given by

$$ds^2 = \sum_{i=1}^{n}(\omega_i)^2.$$

If we take a rectangular frame in the tangent euclidean space at any point of the Riemann space, a Riemannian connection is given by

$$dA = \omega_i e_i, \quad de_i = \omega_{ij} e_j \quad (\omega_{ij} = -\omega_{ji}). \tag{1}$$

If we denote the outer derivative of ω by $d\omega$, we have

$$d\omega_i = [\omega_j \omega_{ji}]. \tag{2}$$

Taking the outer derivative of both sides, we get

$$[\omega_j \Pi_{ji}] = 0 \tag{3}$$

where Π_{ij} is given by

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