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Nagoya Math. J.
Vol. 49 (1973), 155-229

QUALITATIVE THEORY OF CODIMENSION-ONE FOLIATIONS

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Introduction. The object of the present paper is to give a method of studying the topological properties of integral manifolds defined by a completely integrable one-form.

Our method is differential-topological. Through the singular points of the variation equation of the given one-form, we investigate the qualitative properties of the integral manifolds.

The plan of this paper is as follows. In § 1, we state our main theorems, which assert that under certain conditions, foliated structures are classified into three groups, "bundle foliations", "Reeb foliations", "hyperbolic foliations". In § 2, which is one of the most important parts in our theory, we introduce the concept of a vein. A vein is a leaf of a certain codimension-two foliation associated with the given foliation. Proposition 2.2.1 is concerned with the existence of compact veins. In § 3, we study precisely the distance between two leaves along a curve contained in one of them. "Admissible tangential curves" and their "lifts" are the fundamental tools in the proofs of the main theorems. In § 4, we introduce a special Riemannian structure convenient for the proof of Proposition 4.2.1 from which we prove three fundamental lemmas 4.1.2–4.1.4. Using these lemmas, we prove our theorems I, II, III, in §§ 5, 6, 7, respectively. § 8 is devoted to the proofs of Propositions 4.1.1 and 4.2.1. In the appendix, we prove that our condition (T) is "generic".

The main results of this paper have been announced in [8].

The author wishes to express his gratitude for the guidance and encouragement received from Professor Y. Shikata.

Notation. $\boldsymbol{R}$ denotes the field of real numbers, and $\boldsymbol{R}^m$ denotes the real m-space, regarded as a real vector space or as a smooth manifold. S^m denotes an ordinary m-dimensional sphere. By an m-manifold, we

Received September 28, 1972.