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MANIFOLDS WHICH DO NOT ADMIT ANOSOV DIFFEOMORPHISMS

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In [3], M. W. Hirsch obtained some necessary conditions for the existence of an Anosov diffeomorphism on a differentiable manifold. As an application, he constructed many manifolds which do not admit Anosov diffeomorphisms.

In this paper, we shall give a new necessary condition for the existence of an Anosov diffeomorphism on a compact connected differentiable manifold. Our result seems weaker than that of M. W. Hirsch's, but it can be applied for the case in which his results are not applicable. For example, the following manifolds are among those which do not admit Anosov diffeomorphisms: Homotopy spheres, projective spaces over real, complex, and quaternion number fields, and lens spaces.

Throughout this paper "differentiable" means "differentiable of class C^∞" unless otherwise stated.

§ 1. The Main Theorem

Let M be a Riemannian manifold. A C^1-diffeomorphism $f: M \to M$ is called an Anosov diffeomorphism if there exist a continuous splitting

$$T(M) = E^u \oplus E^s$$

of the tangent bundle of M and constants $C > 0, \lambda > 1$ such that $Tf(E^u) = E^u, Tf(E^s) = E^s$, and, furthermore, for all positive integers m and tangent vectors $x \in T(M)$:

$$|Tf^m(x)| \geqq C\lambda^m|x| \qquad \text{if } x \in E^u ,$$
$$|Tf^m(x)| \leqq C^{-1}\lambda^{-m}|x| \qquad \text{if } x \in E^s ,$$

where Tf^m is the tangent of f^m, and $|\cdot|$ is the norm of the tangent vector.

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