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INVARIANTS OF CERTAIN GROUPS I[1]

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Let G be a group and let k be a field. *A* k-representation ρ of G is a homomorphism of G into the group of non-singular linear transformations of some finite-dimensional vector space V over k. Let K be the field of fractions of the symmetric algebra $S(V)$ of V, then G acts naturally on K as k-automorphisms. There is a natural inclusion map $V \to K$, so we view V as a k-subvector space of K. Let $v_1, v_2, \cdots, v_n$ be a basis for V, then K is generated by $v_1, v_2, \cdots, v_n$ over k as a field and these are algebraically independent over k, that is, K is a rational field over k with the transcendence degree n. All elements of K fixed by G form a subfield of K. We denote this subfield by K^G.

We say that ρ has the property $[R]$ *if K^G is a rational field over k.*

Kuniyoshi proved that if G is a finite p-group and if k is a field of characteristic p, the regular representation has the property $[R]$ ([3]). Gaschütz generalized this result to an arbitrary representations ([2]). We shall give other generalizations of their results.

Let G be a group and let ρ be a k-representation of G. Let V be the underlying space of this representation. ρ is called triangularizable if there exisrs a G-invariant flag[3] in V.

Followings are examples of triangularizable representations:

(1) G is a finite commutative group of exponent m and k is a field whose characteristic does not divide m and which contains a primitive m-th root of unity. Then every k-representation of G is triangulariazble.

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3) *A* flag F in V is a sequence of subspaces of V F: $V=V_n \supset V_{n-2} \supset \cdots V_1 \supset V_0=(0)$ such that $\dim V_i = i$ ($n = \dim V$). F is G-invariant if $\rho(g)(V_i) \subset V_i$ for all $g \in G$ and all i.