

ELEMENTARY QUOTIENTS OF ABELIAN GROUPS, AND SINGULAR HOMOLOGY ON MANIFOLDS

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§ 0. Introduction. The topological origin of the problem. Let there be given a compact topological manifold M_n . If M_n admits a “triangulation” it is known that the fundamental invariants, namely the connectivities of M_n over fields, the Betti numbers and torsion coefficients over $\mathbf{Z}$ of the singular homology groups of M_n , are finite and calculable. However it is not known that a “triangulation” of M_n always exists when $n > 3$.

Singular homology groups are understood in the sense of Eilenberg [1].

An alternative to triangulation of M_n . When M_n is differentiable, of at least class C^2 , the alternative to the hypothesis of triangulation will be understood to be the existence of a differentiable non-degenerate (ND) function¹⁾ f on M_n . When M_n is not known to be so differentiable the alternative will be understood to be the existence in the sense of [2] of a topologically non-degenerate (TND) function¹⁾ f on M_n .

We shall be concerned with the subsets

$$(0, 0) \quad f_c = \{p \in M_n \mid f(p) \leq c\}$$

of M_n where c is an arbitrary value of f on M_n and shall term f_c a *sublevel set* of M_n .

In a series of papers which make no use of a triangulation of M_n we shall show that the fundamental invariants of the singular homology groups of the sublevel sets f_c of M_n are uniquely determined by suitable relative numerical invariants associated with the respective critical points on f_c of

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¹⁾ The function f can and will be chosen so as to have different values a at different critical points of f .