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ON ε-ENTROPY OF EQUIVALENT GAUSSIAN PROCESSES

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To Professor Katuzi Ono on the occasion of his 60th birthday

§1. Introduction.

Let $\xi = \{\xi(t);\ t\in T\}$ be a stochastic process, where T is a finite interval. The ε-entropy $H_\varepsilon(\xi)$ of ξ is defined as the following quantity:

$$H_\varepsilon(\xi) = \inf_{\eta} I(\xi, \eta), \tag{1}$$

where $I(\xi,\eta)$ is the amount of information about ξ contained in an another stochastic process $\eta = \{\eta(t);\ t\in T\}$ and the infimum is taken for all stochastic processes η satisfying the condition:

$$\int_T E|\xi(t) - \eta(t)|^2 dt \leqq \varepsilon^2. \tag{2}$$

Concerning the ε-entropy of Gaussian processes, M.S. Pinsker has got an explicit expression of it in terms of the spectral measure. More precisely, let $\xi = \{\xi(t);\ t\in T\}$ be a real valued mean continuous Gaussian process. We denote by $r(s,t)$ the covariance function of ξ, i.e. $r(s,t) = E\{(\xi(s) - E\xi(s))(\xi(t) - E\xi(t))\}$, and we define an integral operator on the space $L^2(T)$ by the following:

$$K\varphi(t) = \int_T r(s,t)\varphi(s)ds, \quad \varphi\in L^2(T),\ \ t\in T. \tag{3}$$

Then K is a symmetric Hilbert-Schmidt operator with countable nonnegative eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$. Using these eigenvalues, the ε-entropy of ξ is expressed in the form:

$$H_\varepsilon(\xi) = \frac{1}{2}\sum_{n=1}^{\infty} \log\left[\max\left(\frac{\lambda_n}{\theta^2},\ 1\right)\right], \tag{4}$$

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