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ON REAL QUADRATIC FIELDS CONTAINING UNITS WITH NORM -1

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Let $\boldsymbol{Q}$ be the rational number field, and let $K=\boldsymbol{Q}(\sqrt{D})$ ($D>0$ a rational integer) be a real quadratic field. Then, throughout this paper, we shall understand by the fundamental unit ε_D of $\boldsymbol{Q}(\sqrt{D})$ the normalized fundamental unit $\varepsilon_D>1$.

Recently H. Hasse investigated variously real quadratic fields with the genus 1, but with the class number more than one[1]. However, since he needed there to know a explicit form of the fundamental unit of a real quadratic field, his investigation had naturally to be restricted within the case of real quadratic fields of Richaud-Degert type whose fundamental units were already given explicitly.

In this paper, we shall give explicitly the fundamental units of real quadratic fields of the more general type than Richaud-Degert's in the case of real quadratic fields with the fundamental unit ε satisfying $N\varepsilon=-1$, and consider the class number of real quadratic fields of this type as Hasse did in the case of Richaud-Degert type.

In §1, by means of expressing any unit $\varepsilon=(t+u\sqrt{D})/2$ of $\boldsymbol{Q}(\sqrt{D})$ as a function of t, we shall give first a generating function of all real quadratic fields with the fundamental unit whose norm is equal to -1 (Theorem 1). In §2, by means of classifying all units $\varepsilon=(t+u\sqrt{D})/2$ with $N\varepsilon=-1$ by the positive value of u, we shall prove that in the class of $u=p$ or $2p$ (p is 1 or prime congruent to 1 mod 4) the unit $\varepsilon=(t+u\sqrt{D})/2>1$ becomes the fundamental unit of $\boldsymbol{Q}(\sqrt{D})$ except for at most finite number of values of D (Theorem 2 and its Corollary). Moreover, we shall show that real quadratic fields of Richaud-Degert type essentially correspond to real quadratic fields with the fundamental unit belonging to the class of $u=1$ or 2 in such classification (Proposition 2). In §3, we shall give an estima-

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1) Cf. H. Hasse [3].