ON SIGNED BRANCHING MARKOV PROCESSES WITH AGE

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To Professor Kiyoshi Noshiro on the occasion of his 60th birthday

§1. **Introduction.** Many authors have considered branching Markov processes for the probabilistic treatment of semi-linear equations. Recently J.E. Moyal [11], [12] gave a formulation for a wide class of branching processes. A similar idea was used in A.V. Skorohod [18] and N. Ikeda-M. Nagasawa-S. Watanabe [4]-[7]. Applying their method, we shall consider in this paper the following problems (A) and (B).

(A): Let E be a compact Hausdorff space with the second axiom of countability and assume the following are given: (1) H_t: a strongly continuous semi-group on $\boldsymbol{C}(E)=\{f;$ continuous function on $E\}$, (2) $\mathscr{G}$: the infinitesimal operator of H_t, (3) $k(x)$, $q_n(x)$, $n=0,1,2,\cdots$, are continuous functions on E such that $k(x)=\sum_{n=0}^{\infty}q_n(x)$ and $\sum_{n=0}^{\infty}|q_n(x)|<\infty$. *How can we interprete probabilistically the following equation?*

$$(1.1)\qquad \frac{\partial u(t,x)}{\partial t}=\mathscr{G}u(t,x)+k(x)F(x;u(t,x)),\qquad x\in E,\ t\geqq 0,$$

where

$$(1.2)\qquad F(x;\xi)=\frac{1}{k(x)}\sum_{n=0}^{\infty}q_n(x)\xi^n,\qquad x\in E,\ \xi\in R^1.$$

(B): *How can we interprete probabilistcally the following equation?*

$$(1.3)\qquad \frac{\partial u(t,x)}{\partial t}=\frac{1}{2}\Delta u(t,x)+G(u(t,x)),\qquad x\in R^d,\ t>0,^{1)}$$

where Δ denotes the Laplacian in x and $G(\xi)$ satisfies

$$(1.4)\qquad G(0)=G(1)=0,\ G(\xi)>0 \text{ and } G'(0)>G'(\xi),\qquad 0<\xi<1.$$

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1) R^d denotes the d–dimensional Euclidian space.