EQUIVALENCE CLASSES OF MAXIMAL ORDERS

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Introduction. Let k denote the quotient field of a complete discrete rank one valuation ring R. The purpose of this paper is to establish a relationship between the Brauer group of k and the set of maximal orders over R which are equivalent to crossed products over tamely ramified extensions of R.

The Brauer group $B(k)$ of k is the union of groups $H^2(G, U(L))$ where L ranges over the set of all finite Galois extensions of k and G denotes the Galois group of L over k (see pp. 206–207 of [2]). The subset $V(k) = \cup H^2(G, U(L))$ where L ranges over all unramified extensions of k forms a subgroup of $B(k)$. In Section 1 we associate to each element of $V(k)$ a positive integer called its Brauer number. Then we define $T(k)$ to be the set of elements of $V(k)$ whose Brauer numbers are relatively prime to the characteristic of $\bar{R}$, and prove that $T(k)$ is a subgroup of $B(k)$. The object of the paper is to prove the following main theorem.

THEOREM. *Let k denote the quotient field of a complete discrete rank one valuation ring R. A maximal order over R in a central simple k-algebra $\sum$ is equivalent to a crossed product over a tamely ramified extension of R if and only if the Brauer class of $\sum$ is in the subgroup $T(k)$ of $B(k)$.*

The method of proof employs the theory of crossed products, and entails the construction of certain wildly ramified Galois extensions of k. For this, a separate treatment of the equicharacteristic case and the case of unequal characteristic is necessary (Sections 2 and 3 respectively).

We obtain as a corollary to the main theorem the fact that if R is an equicharacteristic ring of characteristic zero, then every maximal R-order is equivalent to a crossed product over a tamely ramified extension of R. We then exhibit the existence of a maximal R-order which is not equivalent to a

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