RESIDUE RINGS OF SEMI-PRIMARY HEREDITARY RINGS*

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Introduction: Throughout this paper we assume that all rings contain an identity. We say that R is a semi-primary ring if its (Jacobson) radical N is nilpotent, and R/N is an Artinian ring. We say that R admits a splitting, and we write $R=A+B$ if A is a subring of R, if B is a two-sided ideal in R, and if $A\cap B=0$.

It has been shown in [1] that for a semi-primary ring R $l\cdot gl.\dim R = r\cdot gl.\dim R = 1 + l.\text{proj. dim } N$. This common value is denoted by $gl\cdot\dim R$.

It has been shown in [2] that if R is a semi-primary hereditary ring, and I is a two-sided ideal in R, then $gl\cdot\dim R/I<\infty$.

We prove that if R is a semi-primary ring and $gl\cdot\dim R/N^2<\infty$, then R is a residue ring of a semi-primary hereditary ring. This is a generalization of a similar result in [3]. The crucial step is a splitting theorem that we prove for a semi-primary ring R, for which $eNe=0$ for any primitive idempotent $e\in R$. This splitting theorem seems also useful in studying certain types of semi-primary subrings of a simple ring.

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§1. A Splitting Theorem.

For the rest of this section, let $R=\sum_{u=1}^{t} Re_u$ be a complete decomposition for the semi-primary ring R, i.e. $e_1 ...... e_t$ are primitive orthogonal idempotents (e.g. [4, pp 53–57]). Furthermore, assume $e_v N e_v=0$ for $v=1, ..., t$. When writing $e_i, e_j ......$ we always assume $1\leq i, j, ...\leq t$, unless otherwise stated.

Since for any e_i, e_iNe_i is the radical of e_iRe_i, and e_iRe_i/e_iNe_i is a division ring, we have:

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