ON THE PSEUDO-PERIODS OF THE WEIERSTRASS ZETA FUNCTIONS II*

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To Professor Kiyoshi Noshiro on the occasion of his sixtieth birthday.

1. In a recent note bearing the same title (SIAM Journ. of Numerical Analysis, Vol. 3, 1966), I showed that there existed Weierstrass zeta functions having a vanishing pseudo-period. Cf. line 8, p. 376 of Saks-Zygmund, *Analytic Functions*. Professor Peter Henrici kindly pointed out to me that the mere existence of a Weierstrass zeta function with a vanishing pseudo-period was readily concluded from a classical formula of the theory of elliptic functions (Hurwitz-Courant, *Funktionentheorie*, 5th ed., p. 210) which reduces the question to the study of the zeros of

$$\frac{1}{12}-\sum_{1}^{\infty}\frac{kx^k}{1-x^k}\ ,\quad |x|<1.$$

Here the presence of a zero on $]0, 1[$ is obvious. However a much stronger result was implicit in my note since the argument there employed serves to show that *any ordered pair of complex numbers, not both zero, appears as the pseudo-period pair of some Weierstrass zeta function*, and in fact even more.

The argument of the cited note is based on the study of a system of functional equations associated with the modular group, the use of the modular function λ, and the big Picard theorem. Thus it may be said that the note treated a theme of the classical Weierstrass function theory with methods that draw in some part on ideas stemming from the Weierstrass theory. Professor Krishna Chandrasekharan remarked to me that the study of systems of functional equations of the kind treated in the cited note were of interest for Fuchsian and Fuchsoid groups in general. Actually, certain systems of this more general type were studied in an earlier paper of mine (A generalization of the Aumann-Carathéodory "Starrheitssatz", Duke Math. J. 1941).

In the present paper we study the pseudo-period pairs of the Weierstrass

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