ON A CLASS OF MARKOV PROCESSES TAKING VALUES ON LINES AND THE CENTRAL LIMIT THEOREM

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To Professor Kiyoshi Noshiro on the occasion of his 60-th birthday

§1. Introduction

We shall consider a class of Markov processes $(n(t), x(t))$ with the continuous time parameter $t\in[0, \infty)$, whose state space is $\{1, 2,\dots, N\}\times R^1$. We shall assume that the processes are spacially homogeneous with respect to $x\in R^1$. In detail, our assumption is that the transition function

$$F_{ij}(x, t)=P(n(t)=j, x(t)\leqq x\,|\,n(0)=i, x(0)=0),\quad t>0,\ 1\leqq i, j\leqq N, x\in R^1,$$

satisfies following conditions $(1, 1)\sim(1, 4)$.

(1, 1) $F_{ij}(x, t)$ is non-negative, and, for fixed i, j and t, it is monotone non-decreasing and right continuous in $x\in R^1$.

$$(1, 2)\qquad \begin{aligned}&F_{ij}(+\infty, t)=\lim_{x\to\infty}F_{ij}(x, t)\leqq 1,\\ &F_{ij}(-\infty, t)=\lim_{x\to\infty}F_{ij}(x, t)=0,\qquad 1\leqq i, j\leqq N,\quad t>0,\\ &\sum_{i=1}^{N}F_{ij}(+\infty, t)=1,\qquad i=1, 2,\dots, N,\quad t>0,\end{aligned}$$

$$(1, 3)\qquad F_{ij}(x, t)=\sum_{k=1}^{N}\int_{R^1}F_{ik}(x-y, t)dF_{kj}(y, s)\quad t, s>0,$$
$$1\leqq i, k\leqq N,\quad x\in R^1,$$

$$(1, 4)\qquad \lim_{t\downarrow 0}F_{ij}(x, t)=\begin{cases}\delta_{ij}, & x\in[0, +\infty)\\ 0, & x\in(-\infty, 0).\end{cases}$$

The central limit theorem for processes of this type, in case of the discrete time parameter and in a special case of the continous time parameter, has been obtained by Keilson and Wishart [3]. In this paper, through introducing a system of generators of the semi-groups related to the processes, we show that the central limit theorem is valid for our cases of the continous time parameter.

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