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THETA-FUNCTIONS AND HILBERT MODULAR FORMS

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Introduction

The purpose of this note is to show how the theta-functions attached to certain indefinite quadratic forms of signature (2, 2) can be used to produce a map from certain spaces of cusp forms of Nebentype to Hilbert modular forms. The possibility of making such a construction was suggested by Niwa [4], and the techniques are the same as his and Shintani's [6]. The construction of Hilbert modular forms from cusp forms of one variable has been discussed by many people, and I will not attempt to give a history of the subject here. However, the map produced by the theta-function is essentially the same as that of Doi and Naganuma [2], and Zagier [7]. In particular, the integral kernel $\Omega(\tau, z_1, z_2)$ of Zagier is essentially the 'holomorphic part' of the theta-function.

Professor Asai has kindly informed me that he has also considered the case of signature (2, 2) and has obtained similar results. In [9], Professor Asai has studied the case of signature (3, 1) and has shown that forms of signature (3, 1) can be used to produce a lifting of cusp forms of Neben type to modular forms on hyperbolic 3-space with respect to discrete subgroups of $SL_2(\boldsymbol{C})$. The case of signature $(n-2, 2)$ has been considered by Rallis and Schiffman [10], [11], and by Oda [12].

1. Construction of the theta-functions

Let $k = \boldsymbol{Q}(\sqrt{\Delta})$ be the real quadratic field with discriminant Δ, and let σ be the Galois automorphism of $k/\boldsymbol{Q}$. Let

$$\begin{aligned} V &= \{X \in M_2(k) \text{ such that } X^{\iota} = -X^{\sigma}\} \\ &= \left\{X = \begin{pmatrix} x_1 & x_4 \\ x_3 & -x_1^{\sigma} \end{pmatrix};\ x_1 \in k,\ x_3,\ x_4 \in \boldsymbol{Q}\right\}. \end{aligned}$$

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