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ANOTHER APPROACH TO A NON-ELLIPTIC BOUNDARY PROBLEM

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1. Introduction

Let Ω be a bounded domain in n dimensional Euclidian space $\boldsymbol{R}^n$ $(n \geqq 2)$ with C^∞ boundary Γ of dimension $n-1$ and let there be given two real-valued C^∞-functions α, β on Γ such that $\alpha \geqq 0$, $\beta \geqq 0$ and $\alpha + \beta = 1$ throughout Γ. Then we consider the non-elliptic boundary value problem with $\lambda \geqq 0$ (which is always assumed, and in particular when $\lambda = 0$, we further assume $\beta \not\equiv 0$, throughout this paper):

$$(1) \qquad \begin{cases} (\lambda - \Delta)U = F & \text{in } \Omega \\ \alpha \dfrac{\partial U}{\partial n} + \beta U = 0 & \text{on } \Gamma \end{cases}$$

where $\Delta = (\partial/\partial x_1)^2 + \cdots + (\partial/\partial x_n)^2$ and $\partial U/\partial n$ denotes the exterior normal derivative of U. This kind of problem has been recently discussed from the viewpoint of functional analysis by several authors. In [1], [4] and [5] they used the variational approach in Ω, applying the elliptic regularization, and in [3] and [6] they reduced the problem (1) to a pseudo-differential equation on Γ and applied what is called Melin's theorem.

In this paper, we would like to note that the problem (1) can also be solved without using Melin's theorem in the latter method. Instead of it, we shall apply the classical Riesz-Schauder theory (see Section 7).

2. Operator S

We shall denote by $P\varphi$ the unique solution of the Dirichlet problem

$$(2) \qquad \begin{cases} (\lambda - \Delta)U = 0 & \text{in } \Omega \\ U = \varphi & \text{on } \Gamma\,. \end{cases}$$

Then it follows that the mapping of $C^\infty(\Gamma)$ into itself;

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