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ON THE GENERALIZED TEICHMÜLLER SPACES AND DIFFERENTIAL EQUATIONS

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It is well known that for the family F of Riemann surfaces $\{R(z)\}$ defined by the equations $y^2 = x(x-1)(x-z)$, $z \in C - \{0, 1\}$, we have one independent abelian differential $\omega = y^{-1}dx$ on each $R(z)$ and if we consider z as a parameter on $C - \{0, 1\}$, the integrals $\int_g^h y^{-1}dx$ $(g, h = 0, 1, \infty)$ are solutions of the Gauss's differential equation

$$z(z-1)\frac{d^2w}{dz^2} + (2z-1)\frac{dw}{dz} + \frac{w}{4} = 0 .$$

If we take two suitable solutions $w_1(z), w_2(z)$ of the equation, and denote the ratios of $w_1(z)$ and $w_2(z)$ by τ, then the inverse function $z(\tau)$ is a single valued holomorphic function on the upper-half τ plane.

The first aim of this paper is to consider families of Riemann surfaces for the differential equations of Fuchsian type, just as we have considered the family F for the Gauss's differential equation. These are investigated in §1 and §2. The second aim is to investigate the analytic properties in these families by considering the symmetric domain H which was introduced by Shimura [9] and the generalized Teichmüller space which was constructed in [5]. These are studied in §3. The main result here is Theorem (3.3.7), which is an answer to the extension of the case of the Gauss's differential equation.

Just as for the family F the parameter z is the Lambda function in the upper half plane, and is represented by the quotients of theta constants, our third aim is to similarly investigate the analyticity of the parameters, which we have in 1.3, in the complex structure. We study this problem in §4, wherein an answer is given by Theorem (4.2.11).

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