

## A CORRESPONDENCE BETWEEN QUATERNARY QUADRATIC FORMS

PAUL PONOMAREV\*

### Introduction

Let  $p$  be a prime  $\equiv 1 \pmod{4}$ . Let  $\mathfrak{A}$  denote the quaternion algebra of discriminant  $p^2$  over the field of rational numbers  $\mathcal{Q}$ , and let  $V$  be a quaternary quadratic space over  $\mathcal{Q}$  of discriminant  $p(\mathcal{Q}^\times)^2$ . In this note we establish a natural correspondence between the similitude classes of two-sided normal ideals of  $\mathfrak{A}$  and certain similitude classes of the lattices of  $V$  which have reduced discriminant  $p$  or  $p^3$ . The classes for which it fails to be a function can be explicitly described, and at such classes it is at worst "one-to-two", the two associated classes merely being the duals of each other.

In the classical terminology, our correspondence is between classes of positive definite integral quaternary forms which have an improper automorphism, on the one hand, those of discriminant  $p^2$ , and, on the other hand, those of discriminant  $p$  or  $p^3$ . In both cases the classes having an improper automorphism can be obtained by taking the classes which represent 1 along with their adjoints. In §5 we disprove a conjecture of Hecke ([4], p. 884) concerning the linear independence of the theta series coming from a fixed column of the Brandt matrices associated to  $\mathfrak{A}$ . We propose, instead, that the theta series coming from the classes having an improper automorphism should provide a basis for the corresponding space of modular forms. In the case of Nebentypus this reduces to a conjecture of Kitaoka ([5], p. 152). If the more general conjecture were verified, then our correspondence would have the property of associating a basis of modular forms of Nebentypus  $\left(-2, p, \left(\frac{-}{p}\right)\right)$  to a basis of modular forms of Haupttypus  $(-2, p, 1)$ .

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