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THE CAMERON-STORVICK FUNCTION SPACE INTEGRAL: AN $L(L_p, L_{p'})$ THEORY

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0. Introduction.

In [3] Cameron and Storvick introduced a very general operator-valued function space "integral". In [3–5, 8, 9, 11, 13–20] the existence of this integral as an operator from L_2 to L_2 was established for certain functions. Recently the existence of the integral as an operator from L_1 to L_∞ has been studied [6, 7, 21]. In this paper we study the integral as an operator from L_p to $L_{p'}$ where $1 < p \le 2$. The resulting theorems extend the theory substantially and indicate relationships between the L_2-L_2 and L_1-L_∞ theories that were not apparent earlier. Even in the most studied case, $p = p' = 2$, the results below strengthen the theory.

Before giving the basic definitions, we fix some notation. $\boldsymbol{R}^\nu$ will denote ν-dimensional Euclidean space. $\boldsymbol{C}$ will denote the complex numbers and $\boldsymbol{C}^+$ the complex numbers with positive real part. If Y and Z are Banach spaces, $L(Y, Z)$ will denote the space of continuous linear operators from Y to Z. For ν a positive integer, let $C^\nu[a, b]$ denote the space of $\boldsymbol{R}^\nu$-valued continuous functions on $[a, b]$. $C_0^\nu[a, b]$ will denote those X in $C^\nu[a, b]$ such that $X(a) = 0$. $C_0^\nu[a, b]$ will be referred to as "Wiener space" and integration over $C_0^\nu[a, b]$ will always be with respect to Wiener measure.

Let $1 < p \le 2$ be given. Let F be a function from $C^\nu[a, b]$ to $\boldsymbol{C}$. Given $\lambda > 0$, ψ in $L_p(\boldsymbol{R}^\nu)$ and ξ in $\boldsymbol{R}^\nu$, let

$$(0.1) \qquad (I_\lambda(F)\psi)(\xi) \equiv \int_{C_0^\nu[a,b]} F(\lambda^{-1/2}X + \xi)\psi(\lambda^{-1/2}X(b) + \xi)dm(X) .$$

If $I_\lambda(F)\psi$ is in $L_{p'}(\boldsymbol{R}^\nu)$ as a function of ξ and if the correspondence $\psi \to I_\lambda(F)\psi$ gives an element of $L \equiv L(L_p(\boldsymbol{R}^\nu), L_{p'}(\boldsymbol{R}^\nu))$, we say that the operator-valued function space integral $I_\lambda(F)$ exists.

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