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STOCHASTIC INTEGRALS IN ABSTRUCT WIENER SPACE II: REGULARITY PROPERTIES

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Introduction

This paper continues the study of stochastic integrals in abstract Wiener space previously given in [14]. We will present, among other things, the detailed discussion and proofs of the results announced in [16]. Let $H \subset B$ be an abstract Wiener space. Consider the following stochastic integral equation in $H \subset B$,

$$(1) \qquad X(t) = x + \int_0^t A(s, X(s))dW(s) + \int_0^t \sigma(s, X(s))ds \,,$$

where $W(t)$ is a Wiener process in B. Under certain assumptions on A and σ we showed in [14] that (1) has a unique non-anticipating continuous solution and that this solution is a Markov process. If A and σ are differentiable in the second variable we can differentiate the above equation "formally" with respect to the starting point x to obtain the formal operator-valued stochastic integral equation

$$(2) \qquad Y(t) = I + \int_0^t A_x(s, X(s))Y(s)dW(s) + \int_0^t \sigma_x(s, X(s))Y(s)ds \,,$$

where A_x and σ_x are derivatives of A and σ in the second variable, respectively. (2) is a linear integral equation and obviously has a unique solution which qualifies to be called the derivative of $X(t)$ in some sense. If A and σ are furthermore twice differentiable we can differentiate (2) formally in the same manner to obtain another stochastic integral equation whose solution is the second derivative of $X(t)$. Thus roughly speaking, the solution $X(t)$ of (1), regarded as a function of its starting point, is as smooth as A and σ.

Let f be a real-valued continuous function in B. Let $\theta(x) =$

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