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THE APPLICATION OF THE PRINCIPAL IDEAL THEOREM TO p-GROUPS

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Introduction

Let p be a fixed prime integer, and G a finite p-group. For a subgroup H of G, we denote the centralizer of H in G by $C_G(H)$. The commutator subgroup of G is denoted by $[G, G]$. One of the main results of this paper is

THEOREM 1. *Let A be a normal abelian subgroup of G. Suppose that* (1) *$G/C_G(A)$ is regular, and that* (2) *$\langle g \rangle \cdot A$ is regular for each $g \in G$. Then the exponent of G divides the index $[G: A \cap [G, G]]$.*

Because a p-group of class less than p is regular, we have the following theorem as a corollary: Let

$$K_1(G) = G \supset K_2(G) \supset \cdots \supset K_n(G) \supset \cdots$$

and

$$Z_0(G) = 1 \subset Z_1(G) \subset \cdots \subset Z_{n-1}(G) \subset \cdots$$

be the lower and the upper central series of G, respectively.

THEOREM 2. *Let A be a maximal one among normal abelian subgroups of G which are contained in $Z_{p-1}(G) \cap C_G(K_p(G))$. Then the exponent of G divides $[G: A \cap [G, G]]$.*

If A is as in the theorem, then the center $Z(G) = Z_1(G)$ of G is a subgroup of A. Therefore, the index of the theorem certainly divides $[G: Z(G) \cap [G, G]]$. Hence Theorem 2 is a generalization of the result of Alperin and Tzee-Nan Kuo [1]. Furthermore, it is best possible since the exponent of G coincides with the index $[G: A \cap [G, G]]$ if G is the irregular p-group of Blackburn exhibited by Huppert [3, Ch. III, 10.15] with $A = [G, G]$. In this case, $[G: A] = p^2$ and $[G: Z(G)] = p^p$. (See Ch. II, § 3 for the detail.)

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