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ON A DUAL RELATION FOR ADDITION FORMULAS OF ADDITIVE GROUPS II

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Chapter 2. Sheffer Polynomials

Introduction

This paper is a continuation of our previous memoir [28], hereafter referred to as I, and constitutes the second chapter of this series. As stated in I, our aim in this series is to examine properties of a polynomial sequence with several variables satisfying an addition formula by means of the down-ladder, and to give a generalization of so called classical polynomials. In the present article, we study the two kinds of polynomial sequences:

(i) sequences $s_\alpha(x)$ of polynomials satisfying the identities

$$s_\alpha(x+y) = \sum_{\alpha=\beta+\gamma} s_\beta(x)p_\gamma(y)\,,$$

where $p_\alpha(x)$ is a given sequence of binomial type defined in I,

(ii) doubly indexed sequences $p_\alpha^{[\lambda]}(x)$ of polynomials satisfying

$$p_\alpha^{[\lambda+\mu]}(x+y) = \sum_{\alpha=\beta+\gamma} p_\beta^{[\lambda]}(x)p_\gamma^{[\mu]}(y)\,.$$

In the case of one variable (cf. [11], [22]), the addition formula (i) or (ii) holds for many well known polynomials, for example, Hermite, Laguerre, Euler, Bernoulli, Poisson-Charlier, Krawtchouk, and Stirling polynomials etc.. In Section 8, some of these polynomials are generalized to the case of several variables.

Let us give a brief description of contents of this paper.

Section 1 deals with fundamental properties of a polynomial sequence $s_\alpha(x)$ to satisfy the addition formula (i), that is called a Sheffer set. In this section, we have a relation between the Sheffer set $s_\alpha(x)$ and the polynomial sequence $p_\alpha(x)$ of binomial type. Also, an expansion formula

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