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Nagoya Math. J.
Vol. 96 (1984), 29–39

BOUNDEDNESS OF SINGULAR INTEGRAL OPERATORS OF CALDERON TYPE, III

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§1. Introduction

In this paper we investigate the boundedness of Cauchy kernels. The Cauchy kernel associated with a locally integrable real-valued function $\theta(x)$ is defined by

$$(1) \qquad \mathfrak{C}[\theta](x, y) = (1 + i\theta(y))/\{(x - y) + i(\Theta(x) - \Theta(y))\}\,,$$

where $\Theta(x) = \int_0^x \theta(z)\,dz$. This kernel plays an important role in harmonic analysis on the graph $\{(x, \Theta(x));\ x \in (-\infty, \infty)\}$. For $p > 1$ and a non-negative function $\omega(x)$, let L^p_ω denote the space of functions $f(x)$ with $\|f\|_{p\omega} = \left\{\int_{-\infty}^{\infty} |f(x)|^p \omega(x)\,dx\right\}^{1/p} < \infty$. In the case $\omega(x) \equiv 1$, we write simply L^p and $\|\cdot\|_p$. We say that $\mathfrak{C}[\theta]$ is of type (p, ω) if, for any $f \in L^p_\omega$,

$$(2) \qquad \mathfrak{C}[\theta]f(x) = \lim_{\varepsilon \to 0} \int_{\varepsilon < |x-y| < 1/\varepsilon} \mathfrak{C}[\theta](x, y) f(y)\,dy$$

exists almost everywhere (a.e.) and $\|\mathfrak{C}[\theta]\|_{p\omega} = \sup\{\|\mathfrak{C}[\theta]f\|_{p\omega}/\|f\|_{p\omega};\ 0 < \|f\|_{p\omega} < \infty\} < \infty$. We also write $\|\mathfrak{C}[\theta]\|_p$ in the case $\omega(x) \equiv 1$. We say that $\omega(x)$ satisfies the Muckenhoupt (A_p) condition if

$$(\mathrm{A}_p) \qquad \sup_I (m_I\omega)(m_I\omega^{-1/(p-1)})^{p-1} < \infty\,,$$

where "$\sup_I$" denotes the supremum over all finite intervals I and $m_I\omega = (1/|I|)\int_I \omega(x)\,dx$ ($|I|$: the measure of I). It is well-known that Calderón-Zygmund kernels are of type (p, ω) if $\omega(x)$ satisfies (A_p) ([2]). We shall show that the analogous property is valid for some Cauchy kernels. We say that a locally integrable function $f(x)$ is of bounded mean oscillation if $\|f\|_{\mathrm{BMO}} = \sup_I m_I|f - m_I f| < \infty$. The space BMO of functions of bounded mean oscillation, modulo constants, is a Banach space with norm $\|\cdot\|_{\mathrm{BMO}}$. We show

Received June 17, 1983.