T. Ono
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ON DEFORMATIONS OF HOPF MAPS AND HYPERGEOMETRIC SERIES

TAKASHI ONO

Introduction

Let $\boldsymbol{R}^n$ denote the Euclidean space of dimension $n \geqq 1$ with the standard inner product $\langle x, y\rangle$ and the norm $Nx = \langle x, x\rangle$. We shall denote by $d\omega_{n-1}$ the volume element of the unit sphere $S^{n-1} = \{x \in \boldsymbol{R}^n; Nx = 1\}$ normalized so that the volume of S^{n-1} is 1.

With each continuous map $f: S^{n-1} \to \boldsymbol{R}^m$, we shall associate a function $f^\#(z)$ of a complex variable z by

$$f^\#(z) = \int_{S^{n-1}} e^{z(f(x))} d\omega_{n-1}\,.$$

Clearly $f^\#(z)$ is an entire function and its Taylor expansion is given by

$$f^\#(z) = \sum_{k=0}^{\infty} N_k(f) \frac{z^k}{k!}$$

where

$$N_k(f) = \int_{S^{n-1}} N(f(x))^k d\omega_{n-1}\,.$$

When f is spherical, i.e. when f maps S^{n-1} in S^{m-1}, we have $f^\#(z) = e^z$. When we are given a family $\{f_t\}$, $0 \leqq t \leqq 1$, of maps: $S^{n-1} \to \boldsymbol{R}^m$ such that f_0 is spherical, we have a family $\{f_t^\#\}$ of entire functions beginning with $f_0^\# = e^z$ and ending with some advanced function $f_1^\#$.

Here is an illustrative example: consider the family

$$f_t(x) = (x_1^2 - x_2^2, 2(1 + t)^{1/2} x_1 x_2)\,, \qquad 0 \leqq t \leqq 1\,.$$

The map f_0: $S^1 \to \boldsymbol{R}^2$ is spherical since it is the squaring $x \mapsto x^2$ in $\boldsymbol{C} = \boldsymbol{R}^2$. Passing to the polar coordinates, we have $N(f_t(x)) = 1 + \mathrm{t} \sin^2 2\theta$ and

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