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SYMMETRIC HOMOGENEOUS CONVEX DOMAINS

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Introduction

Let D be a convex domain in the n-dimensional real number space $\boldsymbol{R}^n$, not containing any affine line and $A(D)$ the group of all affine transformations of $\boldsymbol{R}^n$ leaving D invariant. If the group $A(D)$ acts transitively on D, then the domain D is said to be *homogeneous*. From a homogeneous convex domain D in $\boldsymbol{R}^n$, a homogeneous convex cone $V = V(D)$ in $\boldsymbol{R}^{n+1} = \boldsymbol{R}^n \times \boldsymbol{R}$ is constructed as follows (cf. Vinberg [11]):

$$V(D) = \{(tx, t) \in \boldsymbol{R}^n \times \boldsymbol{R};\ x \in D,\ t > 0\}, \tag{0.1}$$

which is called the *cone fitted on* the convex domain D. Let $G(V)$ be the group of all linear automorphisms of V and g_V the canonical $G(V)$-invariant Riemannian metric on V (cf. e.g. [8]). Then a natural imbedding

$$\sigma\colon\ x \in D \longrightarrow (x, 1) \in V(D) \tag{0.2}$$

is equivariant with respect to the groups $A(D)$ and $G(V)$. Therefore, the Riemannian metric $g_D = \sigma^* g_V$ on D induced from (V, g_V) by σ is $A(D)$-invariant. The Riemannian metric g_D is called the *canonical metric* of D. We note that the canonical metric g_D is given from the characteristic function φ_V of V as follows: Let us put $\varphi_D = \varphi_V \circ \sigma$. Then

$$g_D = \sum_{1 \leq i,j \leq n} \frac{\partial^2 \log \varphi_D}{\partial x^i \partial x^j} dx^i dx^j\,, \tag{0.3}$$

where $(x^1, x^2, \cdots, x^n)$ is a system of affine coordinates of $\boldsymbol{R}^n$.

The purpose of the present paper is to determine (up to affine equivalence) all homogeneous convex domains which are Riemannian symmetric with respect to the canonical metric. The main result obtained is stated as follows. *Every symmetric homogeneous convex domain is affinely equiv-*

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