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TRANSIENT MARKOV CONVOLUTION SEMI-GROUPS AND THE ASSOCIATED NEGATIVE DEFINITE FUNCTIONS

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Dedicated to Professor Makoto Ohtsuka on the occasion of his 60th birthday

§1. Let X be a locally compact and σ-compact abelian group and $\hat{X}$ be the dual group of X[1]. We denote by ξ a fixed Haar measure on X and by $\hat{\xi}$ the Haar measure on $\hat{X}$ associated with ξ. It is well-known that (see, for example, [1]):

(A) For a sub-Markov convolution semi-group $(\alpha_t)_{t\geqq 0}$ on X, there exists a uniquely determined negative definite function ψ on $\hat{X}$ such that

$$\hat{\alpha}_t(\hat{x}) = \exp(-t\psi(\hat{x})) \qquad \text{for any } \hat{x} \in \hat{X} \ (t \geqq 0)\,, \tag{1.1}$$

where $\hat{\alpha}_t$ denotes the Fourier transform of α_t.

(B) For a negative definite function ψ on $\hat{X}$, there exists a uniquely determined sub-Markov convolution semi-group $(\alpha_t)_{t\geqq 0}$ on X satisfying (1.1).

In this case, ψ is called the negative definite function associated with $(\alpha_t)_{t\geqq 0}$.

There is an interesting characterization of the transience of sub-Markov convolution semi-groups.

THEOREM. *Let $(\alpha_t)_{t\geqq 0}$ be a sub-Markov convolution semi-group on X and ψ be the negative definite function associated with $(\alpha_t)_{t\geqq 0}$. Then $(\alpha_t)_{t\geqq 0}$ is transient if and only if* $\mathrm{Re}(1/\psi)$ *is locally $\hat{\xi}$-summable, where* $\mathrm{Re}(1/\psi)$ *denotes the real part of* $1/\psi$.

The "only if" part is easily seen (see, for example, [1]). But it is known only to show the "if" part by probabilistic methods (see [3]).

The purpose of this note is to give a simple and non-probabilistic proof of the "if" part.

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1) We denote by $+$ the product of X and that of $\hat{X}$.