M. Herrmann and S. Ikeda
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REMARKS ON LIFTING OF COHEN-MACAULAY PROPERTY

MANFRED HERRMANN* AND SHIN IKEDA

Let (R, m) be a local noetherian ring and I a proper ideal in R. Let $\mathscr{R}(I)$ be the Rees-ring $\oplus_{n\geq 0} I^n$ with respect to I. In this note we describe conditions for I and R in order that the Cohen-Macaulay property (C-M for short) of R/I can be lifted to R and $\mathscr{R}(I)$, see Propositions 1.2, 1.3 and 1.4.

§ 1. Preliminaries, examples and results

The statements in the following proposition are well known. We give here a short proof.

PROPOSITION 1.1. *For a prime ideal $p \subset R$ let R_p be regular and p/p^2 flat over R/p. If R/p is* C-M *then R is a* C-M *domain and $\mathscr{R}(p^\tau)$ is* C-M *for all $\tau \geqq 1$.*

Proof. By assumption p is generated by a regular sequence (see [HSV], Lemma 3.17, p. 75), in particular we have $\dim R = \dim R/p + \operatorname{ht}(p)$. Therefore by [D] and [HSV], p. 72 R is a domain. Then the C-M property of R/p can be used to get a regular sequence with $\dim R$ elements in R, so R is C-M. Hence by [V] we know that $\mathscr{R}(p^\tau)$ is C-M for all $\tau \geqq 1$.

The statement of Proposition 1.1 is false if the regularity of R_p is replaced by the C-M property. Here is an example of Hesselink (see [HSV], p. 76): Let S be a discrete valuation ring and t a generator of its maximal ideal. Take the ideal

$$J = (X^2, XY - tZ^2, XZ^2, Z^4)$$

in the polynomial ring $H = S[X, Y, Z]$. Then we consider the local ring

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