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ON THE NIWA-SHINTANI THETA-KERNEL LIFTING OF MODULAR FORMS

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Modular forms of half-integral weight are of intrinsic interest: many of the functions of classical number theory transform under a matrix group with half-integral weight. The aim of this paper is to refine some results and techniques which have been introduced to study these functions and the arithmetic information which they contain.

Our results will be most clear if we give a very brief history of the subject. The general theory of modular forms of half-integral weight is a fairly recent development. Although Hecke [4] did some work in the area, in a sense the subject really begins with Shimura's 1973 paper, "On modular forms of half-integral weight" [11].

Shimura demonstrated an extraordinary 'lifting' property for modular forms of half-integral weight. By considering Euler products associated to eigenfunctions of Hecke operators, Shimura constructs a family of maps taking *cusp* forms of half-integral weight to *holomorphic* forms of even, integral weight, which is where the subject has been most studied and best understood. This 'lifting', moreover, takes eigenfunctions to eigenfunctions.

While Shimura proves the lifted functions to be modular forms, he does not completely determine the *level* at which they transform. However, he makes the following conjecture: if the original function transforms at level $4N$, then the lifted form transforms at level $2N$.

Shimura also proves that the lifted forms are in fact *cusp* forms, if the half-integral weight is $\geq 5/2$. The remaining case, weight $3/2$ (weight $1/2$ does not come under consideration), is more complicated. Certain forms, namely the 'theta functions', *fail* to lift to cusp forms. Shimura here conjectures that everything in the 'orthogonal complement' (with respect to the Petersson inner product) *does* lift to a cusp form.

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