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MODULAR FORMS OF DEGREE n AND REPRESENTATION BY QUADRATIC FORMS II

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Let $S^{(m)}$, $T^{(n)}$ be positive definite integral matrices and suppose that T is represented by S over each p-adic integer ring Z_p. We proved arithmetically in [3] that T is represented by S over Z provided that $m \geq 2n + 3$ and the minimum of T is sufficiently large. This guarantees the existence of at least one representation but does not give any asymptotic formula for the number of representations. To get an asymptotic formula we must employ analytic methods. As a generating function of the numbers of representations we consider the theta function

$$\theta(Z) = \sum_{G \in M_{m,n}(Z)} \exp(2\pi i\sigma(S[G]\cdot Z)),$$

where $Z^{(n)} = X + iY = Z'$, $\operatorname{Im} Z = Y > 0$, and σ denotes the trace. Put $N(S, T) = \sharp\{G \in M_{m,n}(Z) | S[G] = T\}$; then we have

$$\theta(Z) = \sum_T N(S, T) \exp(2\pi i\sigma(TZ)).$$

$\theta(Z)$ is a modular form of degree n and we decompose $\theta(Z)$ as $\theta(Z) = E(Z) + g(Z)$, where $E(Z)$ is the Siegel's weighted sum of theta functions for quadratic forms in the genus of S. Put

$$E(Z) = \sum a(T) \exp(2\pi i\sigma(TZ)),$$
$$g(Z) = \sum b(T) \exp(2\pi i\sigma(TZ)).$$

Then $a(T)$, $T > 0$, is given by

$$\pi^{n(2m-n+1)/4} \prod_{k=0}^{n-1} \Gamma((m-k)/2)^{-1} |S|^{-n/2} |T|^{(m-n-1)/2} \prod_p \alpha_p(T, S),$$

and it is easy to see that the constant term of $g(Z)$ vanishes at every cusp. Now it may be expected that

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