S. Greco Nagoya Math. J. Vol. 87 (1982), 95-100

A NOTE ON UNIVERSALLY CATENARY RINGS

SILVIO GRECO

Introduction

The following two related problems in Commutative Algebra are certainly very popular:

- 1. Study the *permanence* of properties from a ring A to an idealadic completion of A; and
- 2. Study the *lifting* of properties from A/I to A where A is a ring which is complete and separated in the *I*-adic topology.

In recent years much effort has been devoted to the study of the permanence and lifting of many properties related to the theory of excellent rings, with special reference to the formal fibers and the openness of loci (see [6] for informations and bibliography).

As for the property UC (universally catenary) the permanence has been proved long since by Seydi (see [11], 1.2), while the only result we know related to the lifting is in [12], 1.1.2, (see 1.7(ii) below for the statement).

The present note contains some contributions to the problem of lifting for the UC property. The main fact is a counterexample (see 1.1), obtained by employing a technique used by Nagata to construct a non UC ring (see either [7], p. 203, ex. 2, or [6], p. 87, (14.E)).

This answers a question posed to the author by H. Matsumura, and shows that the main result of Rotthaus [10] on the lifting of "quasiexcellent" for semilocal rings does not hold for "excellent".

The paper ends with a positive result (see 2.3), namely: the lifting of UC holds if A is assumed to be catenary. This is proved by elementary techniques as a consequence of a theorem of Ratliff [9].

CONVENTIONS. All rings are assumed to be commutative and noetherian.

Received June 24, 1980.

This work was done while the author was a Visiting Professor at Nagoya University, with the financial support of J.C.P.S.

The author is a member of CNR-GNSAGA.