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AN ELIMINATION THEOREM OF UNIQUENESS CONDITIONS IN THE INTUITIONISTIC PREDICATE CALCULUS

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This paper is a sequel to Motohashi [4]. In [4], a series of theorems named "elimination theorems of uniqueness conditions" was shown to hold in the classical predicate calculus LK. But, these results have the following two defects: one is that they do not hold in the intuitionistic predicate calculus LJ, and the other is that they give no nice axiomatizations of some sets of sentences concerned. In order to explain these facts more explicitly, let us introduce some necessary notations and definitions. Let L be a first order classical predicate calculus LK or a first order intuitionistic predicate calculus LJ. n-ary formulas in L are formulas $F(\bar{a})$ in L with a sequence $\bar{a}$ of distinct free variables of length n such that every free variable in F occurs in $\bar{a}$. Sometimes, we shall omit the sequence $\bar{a}$ in an n-ary formula $F(\bar{a})$ if no confusions are likely to occur. Also, an n-ary predicate symbol R is frequently identified with the n-ary formula $R(\bar{a})$. (If necessary, we can assume that $\bar{a}$ is the sequence of first n free variables in a fixed enumeration of the free variables.) If $A(\bar{a}, a)$ and $E(\bar{a}, \bar{b})$ are $(n+1)$-ary formula and $2n$-ary formula, then the existence condition of A, denoted by $\mathrm{Ex}\,A(\bar{a}, b)$ or $\mathrm{Ex}\,A$, is the sentence; $\forall\bar{x}\exists yA(\bar{x}, y)$, the uniqueness condition of A with respect to E, denoted by $\mathrm{Un}\,(A(\bar{a}, b); E(\bar{a}, \bar{b}))$ or $\mathrm{Un}\,(A; E)$, is the sentence; $\forall\bar{x}\forall\bar{y}\forall x\forall y(E(\bar{x}, \bar{y}) \wedge A(\bar{x}, x) \wedge A(\bar{y}, y). \supset x = y)$, and the congruence condition of A with respect to E, denoted by $\mathrm{Co}\,(A(\bar{a}, b); E(\bar{a}, \bar{b}))$ or $\mathrm{Co}\,(A; E)$, is the sentence; $\forall\bar{x}\forall\bar{y}(E(\bar{x}, \bar{y}) \supset \forall x(A(\bar{x}, x) \equiv A(\bar{y}, x)))$. If $E(\bar{a}, \bar{b})$ is the formula $a_1 = b_1 \wedge \cdots \wedge a_n = b_n$, then $\mathrm{Un}\,(A; E)$ and $\mathrm{Co}\,(A; E)$ are written by $\mathrm{Un}\,A$ and $\mathrm{Co}\,A$, respectively. Note that $\mathrm{Co}\,A$ is provable in LJ. Let P be an m-ary predicate symbol. Then P-positive (P-negative) formulas are formulas which have no negative (positive) occurrences of P (cf. Takeuti [9]). P-positive formulas have the

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