Y. Miyahara
Nagoya Math. J.
Vol. 81 (1981), 177-223

INFINITE DIMENSIONAL LANGEVIN EQUATION AND FOKKER-PLANCK EQUATION

YOSHIO MIYAHARA

§ 0. Introduction

Stochastic processes on a Hilbert space have been discussed in connection with quantum field theory, theory of partial differential equations involving random terms, filtering theory in electrical engineering and so forth, and the theory of those processes has greatly developed recently by many authors (A. B. Balakrishnan [1, 2], Yu. L. Daletskii [7], D. A. Dawson [8, 9], Z. Haba [12], R. Marcus [18], M. Yor [26]).

The most basic concept arising there is the so-called *cylindrical Brownian motion*, abbr. c.B.m., (see Definition 1.2). It is thought of as a natural generalization of a finite dimensional Brownian motion, and it can be formed from multi-parameter white noise as is briefly illustrated in what follows.

First we introduce a (Gaussian) white noise μ indexed by a space-time parameter running through $D \times T$, where D is a domain of the d-dimensional Euclidean space $\boldsymbol{R}^d$ and T is $\boldsymbol{R}^1$ on which the time t runs. Namely, μ is the standard Gaussian measure on $\mathscr{E}^*$ determined by the characteristic functional

$$C_\mu(\eta) = \exp\left\{-\frac{1}{2}\|\eta\|^2\right\}, \quad \|\eta\|^2 = \int_{D\times T}|\eta|^2 dx, \quad \eta \in \mathscr{E} ,$$

where $\mathscr{E}^*$ is the dual of $\mathscr{E}$ forming a Gelfand triple:

$$\mathscr{E}^* \subset \mathscr{H} = L^2(D \times T) \subset \mathscr{E}^* .$$

We are now given a generalized stochastic process in the sense that $\langle \eta, \omega\rangle$, $\eta \in \mathscr{E}$, $\omega \in \mathscr{E}^*$, is an ordinary random variable, where $\langle\ ,\ \rangle$ is the canonical bilinear form connecting $\mathscr{E}$ and $\mathscr{E}^*$ (I.M. Gelfand and N. Ya. Vilenkin [11]). The bilinear form $\langle\ ,\ \rangle$ extends to the case where η is of the form $\xi \otimes \chi_{[s,t]}$

Received October 25, 1979.