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THE CANONICAL MODULES OF GRADED RINGS DEFINED BY GENERIC MATRICES

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Let k be a field, and $X = [x_{ij}]$ be an $n \times (n + m)$ matrix whose elements are algebraically independent over k.

We shall study the canonical module of the graded ring R, which is a quotient ring of the polynomial ring $A = k[X]$ by the ideal $\mathfrak{a}_n(X)$ generated by all the $n \times n$ minors of X.

Nowadays it is not difficult to describe the free resolution of the canonical module of R, since the resolution of R as an A module is already known [Eagon-Northcott]. But we will be at a loss for an answer when we are asked what are the generators of the ideal of R which is isomorphic to the canonical module of R, for in most cases the canonical modules of Cohen-Macaulay rings are isomorphic to their ideals, which we might call the canonical ideals.

For this reason, in this paper we shall write down the generators of the canonical ideal of R.

§1. The canonical modules of graded rings

In this section by a graded ring we understand a commutative ring R with a family $\{R_n\}_{n \geqslant 0}$ such that;

(i) $R = \oplus R_n$, $R_n \cdot R_m \subset R_{n+m}$ for all n, $m \geqslant 0$.

(ii) $R_0 = k$ is a field.

(iii) R is finitely generated over k.

Here we shall recall some fundamental facts about the canonical modules of graded rings, all of which are based on the paper of Goto-Watanabe [G-W].

Notation from [G-W]: For any graded ring R, we denote by $M_H(R)$ the category of all the graded R-modules and their homomorphisms of degree 0. Moreover we use the following symbols:

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