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Nagoya Math. J.
Vol. 71 (1978), 87-90

A REMARK CONCERNING THE 2-ADIC NUMBER FIELD

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1. Introduction

Let Q_2 be the 2-adic number field, T/Q_2 be a finite unramified extension, ζ_ν be a primitive 2^ν-th root of unity, and let $K_\nu = T(\zeta_\nu)$. In a previous paper [1, Theorem 11], we stated the following theorem without its proof.

THEOREM A. *Let $R = T(\zeta_\nu + \zeta_\nu^{-1})$, and let σ be a generator of the cyclic Galois group $G(R/T)$. Assume $\nu \geqq 3$. If $N_{R/T}\varepsilon = 1$ for $\varepsilon \in U_R^{(4)}$, then*

$$\varepsilon \in (N_{K_\nu/R}K_\nu^\times)^{\sigma-1} ,$$

where $U_R^{(i)}$ denotes the i-th unit group of R.

The aim of the present paper is to prove this theorem, which is a detailed version of Hilbert's theorem 90 in the 2-adic number field.

2. Preliminaries

Let $\theta = \zeta_\nu + \zeta_\nu^{-1}$. Since $1 - \zeta_\nu$ is a prime element of K_ν,

$$N_{K_\nu/R}(1 - \zeta_\nu) = (1 - \zeta_\nu)(1 - \zeta_\nu^{-1}) = 2 - \theta$$

is a prime element of R. Set $\pi = 2 - \theta$ and denote by ν_π the normalized exponential valuation of R. The Galois group $G(K_\nu/T)$ is isomorphic to the group of prime residue classes mod 2^ν, and hence we can choose the generator σ of $G(R/T)$ such that

$$\theta^\sigma = (\zeta_\nu + \zeta_\nu^{-1})^\sigma = \zeta_\nu^5 + \zeta_\nu^{-5} = \theta^5 - 5\theta^3 + 5\theta ,$$

without loss of generality. Then

$$\pi^\sigma = \pi^5 - 10\pi^4 + 35\pi^3 - 50\pi^2 + 25\pi . \tag{1}$$

LEMMA 1. *Notation being as above, if $\nu \geqq 3$, then*

Received June 10, 1977.