H. Naito
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ON A LOCAL HÖLDER CONTINUITY FOR A MINIMIZER OF THE EXPONENTIAL ENERGY FUNCTIONAL

HISASHI NAITO†

0. Introduction

Let $\Omega \subset \mathbf{R}^m$ be a bounded domain with smooth boundary, where $m \geq 2$. We consider the exponential energy functional

$$E(u) := \int_\Omega e^{|\nabla u|^2} dx \tag{0.1}$$

for $u : \Omega \to \mathbf{R}^n$, where $n \geq 2$.

For minima of certain functionals with suitable growth conditions, many authors have established regularity results [1,5,7,12]. For example, in [7], Hardt and Lin proved that mappings minimizing the L^p-norm of the gradient between compact Riemannian manifolds are smooth except singular sets with finite $(\dim M - [p] - 1)$-dimensional Hausdorff measure.

If a functional has sufficiently rapid growth, we can expect full regularity of minima. In fact, quite recently, Duc and Eells [2], Eells and Lemaire [3] show that, in the case of $n = 1$, a minimizer u of E satisfies $u \in C^\infty(\Omega)$ for any smooth boundary data provided that $\Omega \subset \mathbf{R}^m$ is a strictly convex bounded domain.

In this paper, we consider a local regularity of minima of E for the case of $n \geq 2$. Our main theorem is stated as follows:

(0.2) THEOREM. *For a given boundary data* $g \in \bigcap_{1<p<\infty} W^{1-1/p,p}(\partial\Omega, \mathbf{R}^n)$, *there exists a unique minimizer of* E *on the space* $\{u \in \bigcap_{1<p<\infty} W^{1,p}(\Omega, \mathbf{R}^n) : u|_{\partial\Omega} = g\}$. *If* $B_R(a) \subset \Omega$, *then there exists a number* μ *satisfying* $0 < \mu < 1$ *such that the minimizer belongs to* $C^{1,\mu}(B_{R/4}(a))$.

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