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AUTOMORPHIC FORMS AND INFINITE MATRICES

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In the present paper, we show that an infinite dimensional vector whose components are Fourier coefficients of an automorphic form is characterized as an infinite dimensional vector which is annihilated by an infinite matrix constructed by the values of a Bessel function. Results and methods are all simple and concrete.

Although the idea in the present paper is applicable to more general cases, our investigation will be restricted to the case of automorphic forms of weight 0, i.e., automorphic functions, with respect to $SL(2, \mathbf{Z})$ on the upper half plane, in order to explain the main idea distinctly.

The main theorem is stated in connection with cusp forms in Section 3. In Section 4, we note first that the main theorem is a characterization of eigenvalues of the Laplacian, and show that the eigenvalues of the Laplacian and the zeros of the Riemann zeta function are characterized simultaneously by extending the range of functions slightly out of cusp forms. This fact can be regarded as an example of direct and naive contacts between infinite matrices and zeros of zeta functions.

The most essential part of the present paper consists of approximation formulas proved in Section 2. Because of such formulas, the condition for the components of an infinite vector to be Fourier coefficients of an automorphic form is, as seen for instance in the main theorem, reduced to the sole assertion that the vector is annihilated by an infinite matrix.

§ 1. Preliminaries

In this section, we recall some basic notions, prepare symbols, and prove Theorem 1.1. The main theorem of the present paper (Theorem 3.1) is the converse of Theorem 1.1.

In the sequel, an automorphic form means a function on the upper half plane $S = \{z \in \mathbf{C}\,;\, \mathrm{Im}\, z > 0\}$ which is invariant under the linear transformation

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