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Nagoya Math. J.
Vol. 126 (1992), 125–140

DIFFEOMORPHISMS WITH PSEUDO ORBIT TRACING PROPERTY

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We shall discuss a differentiable invariant that arises when we consider a class of diffeomorphisms having the pseudo orbit tracing property (abbrev. POTP) .

Let M be a closed C^∞ manifold and $\mathrm{Diff}^1(M)$ be the space of diffeomorphisms of M endowed with the C^1 topology. We denote $\mathcal{P}^1(M)$ the C^1 interior of the set of all diffeomorphisms having POTP belonging to $\mathrm{Diff}^1(M)$. Recently Aoki [1] proved that the C^1 interior of the set of all diffeomorphisms whose periodic points are hyperboric, $\mathcal{F}^1(M)$, is characterized as Axiom A diffeomorphisms with no-cycle. After this Moriyasu [8] showed that $\mathcal{P}^1(M) \subset \mathcal{F}^1(M)$ and if $\dim M = 2$ then every $f \in \mathcal{P}^1(M)$ satisfies strong transversality.

In this paper the following two theorems will be proved.

THEOREM A. *There exists a closed C^∞ 3-manifold M such that set of all diffeomorphisms having POTP is not dense in* $\mathrm{Diff}^1(M)$.

The Theorem answers to a problem stated in Morimoto [7].

THEOREM B. *If M is a closed C^∞ 3-manifold, then $\mathcal{P}^1(M)$ is characterized as* Axiom A *diffeomorphisms satisfying strong transversality.*

A diffeomorphism f of M is *quasi-Anosov* if the fact that $\|Df^n(v)\|$ is bounded for all $n \in \mathbf{Z}$ implies that $v = 0$. Theorem A is easily obtained in combining with Franks and Robinson [2] and Sakai [12]. The set of all quasi-Anosov diffeomorphisms belonging to $\mathrm{Diff}^1(M)$, $\mathrm{QA}^1(M)$, is open and $\mathrm{QA}^1(M) \subset \mathcal{F}^1(M)$. It is easy to see that when $\dim M = 2$, every $f \in \mathrm{QA}^1(M)$ is Anosov (see [5]) . However an example of a diffeomorphism f' on the connected sum M' of two 3-tori that is quasi-Anosov but not Anosov was given in Franks and Robinson [2]. Since f' is Ω-stable, there is C^1 neighborhood $\mathcal{U}$ of f' in $\mathrm{Diff}^1(M')$ such that every $g \in \mathcal{U}$ is quasi-Anosov but not Anosov. Thus, by [12] every $g \in \mathcal{U}$ cannot have POTP,

Received February 21, 1991.