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SOME TYPES OF REGULARITY FOR THE DIRICHLET PROBLEM

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The question of whether the existence of a harmonic majorant in a relative neighbourhood of each point of a boundary of a domain D implies the existence of a harmonic majorant in the whole of D has received great attention in recent years and has been dealt with by several authors in different settings. The most general results to date have been achieved in [10] with the Martin boundary. In [9], the author arrives, by independent means, at the conclusions of [10] in the particular case where D is a Lipschitz domain.

In this paper, we answer the question in domains with suitably regular topological frontiers. Our methods rely heavily on the possibility of obtaining an extented-representation for nonnegative superharmonic functions defined near a frontier point. This naturally led to the introduction and the study of new types of regularity for the generalised Dirichlet problem. As well as their suitability in dealing with the question of harmonic majorisation, they present an intrinsic importance as natural extensions of the (classical) regularity. For simplicity reasons, we will treat the finite boundary points and the point at infinity separately.

We start with a type of regularity which, although introduced in a new way, will later be seen to be equivalent to Armitage's strong regularity given in [2].

We first give some conventions concerning the notations.
Unless we specify otherwise, all the sets considered are subsets of N-dimensional Euclidean space R^N with $N \geq 2$.
Points of R^N as well as singletons (i.e. sets consisting of one point) are denoted by a single letter. However, points are, when necessary, expressed in terms of their coordinates. The norm $|\cdot|$ is the Euclidean norm.

For a point y of R^N and a positive reel number r, the open ball $B(y, r)$ is the set

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