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Nagoya Math. J.
Vol. 116 (1989), 17-24

HOMOGENEOUS LINE BUNDLES OVER A TOROIDAL GROUP

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§ 0. Introduction

A connected complex Lie group without non-constant holomorphic functions is called a toroidal group ([5]) or an (H, C)-group ([9]). Let X be an n-dimensional toroidal group. Since a toroidal group is commutative ([5], [9] and [10]), X is isomorphic to the quotient group C^n/Γ by a lattice of C^n. A complex torus is a compact toroidal group. Cousin first studied a non-compact toroidal group ([2]).

Let L be a holomorphic line bundle over X. L is said to be homogeneous if T_x^*L is isomorphic to L for all $x \in X$, where T_x is the translation defined by $x \in X$. It is well-known that if X is a complex torus, then the following assertions are equivalent:

(1) L is topologically trivial.

(2) L is given by a representation of Γ.

(3) L is homogeneous.

But this is not always true for a toroidal group. Vogt showed in [11] that every topologically trivial holomorphic line bundle over X is homogeneous if and only if $\dim H^1(X, \mathcal{O}) < \infty$ ([6]). The cohomology groups $H^p(X, \mathcal{O})$ were classified by Kazama [3] and Kazama-Umeno [4].

In this paper we shall show the equivalence of conditions (2) and (3). In the case that X is a complex torus, a similar equivalence was proved for a vector bundle ([7] and [8]). We state our theorem.

THEOREM. *Let $X = C^n/\Gamma$ be a toroidal group. Then every homogeneous line bundle over X is given by a 1-dimensional representation of Γ.*

The converse of the above theorem is easily seen by the definitions ([11, Proposition 6]). We shall prove the theorem by virtue of the following proposition.

PROPOSITION. *Every homogeneous line bundle over a toroidal group is topologically trivial.*

Received July 27, 1987.