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A REMARK ON THE SPACE OF TESTING RANDOM VARIABLES IN THE WHITE NOISE CALCULUS

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Dedicated to Professor Takeyuki Hida on the occasion of his sixtieth birthday

§ 1. Introduction

The first author and S. Takenaka introduced the structure of a Gel'fand triplet $\mathscr{H} \subset (L^2) \subset \mathscr{H}^*$ into Hida's calculus on generalized Brownian functionals [4–7]. They showed that the space $\mathscr{H}$ of testing random variables has nice properties. For example, $\mathscr{H}$ is closed under multiplication of two elements in $\mathscr{H}$, each element of $\mathscr{H}$ is a continuous functional on the basic space $\mathscr{E}^*$, in addition it can be considered as an analytic functional, and moreover $\exp[t\Delta_V]$ (Δ_V is Volterra's Laplacian) is real analytic in $t \in \boldsymbol{R}$ as a one-parameter group of operators on $\mathscr{H}$, etc.

In this paper, we will prove, by a method different from [4–7], that each element of $\mathscr{H}$ is continuous on the basic space $\mathscr{E}^*$ and by using this result we will show that the evaluation map δ_x: $\varphi \mapsto \varphi(x)$ $(x \in \mathscr{E}^*)$ belongs to $\mathscr{H}^*$. The norm of δ_x will also be estimated.

The fact that δ_x belongs to $\mathscr{H}^*$ is very useful in the argument of positive functionals [8].

§ 2. Gel'fand triplets

Here we will summarize fundamental facts about three Gel'fand triplets $\mathscr{F} \subsetneqq \mathscr{F}^{(0)} \subsetneqq \mathscr{F}^*$, $\exp[\hat{\otimes}\mathscr{E}] \subsetneqq \exp[\hat{\otimes}E_0] \subsetneqq \exp[\hat{\otimes}\mathscr{E}^*]$ and $\mathscr{H} \subsetneqq (L^2) \subset \mathscr{H}^*$, which were introduced and discussed in [4–7, 9], for later use. Let T be a separable topological space with a topological Borel field $\mathscr{B}$ and ν be a σ-finite measure on T without atoms. We suppose that there exists a Gel'fand triplet (or a rigged Hilbert space) $\mathscr{E} \subset L^2(T, \nu) \subset \mathscr{E}^*$ (cf. [3]). Namely, the space $\mathscr{E}$ of testing functions on T is topologized by the pro-

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