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ON F-INTEGRABLE ACTIONS OF THE RESTRICTED LIE ALGEBRA OF A FORMAL GROUP F IN CHARACTERISTIC $p>0$

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§1. Introduction

Let k be an integral domain, let $F=(F_1(X, Y), \cdots, F_n(X, Y))$, $X=(X_1, \cdots, X_n)$, $Y=(Y_1, \cdots, Y_n)$, be an n-dimensional formal group over k, and let $L(F)$ be the Lie algebra of all F-invariant k-derivations of the ring of formal power series $k[\![X]\!]$ (cf. §2). If A is a (commutative) k-algebra and $\mathrm{Der}_k(A)$ denotes the Lie algebra of all k-derivations $d\colon A \to A$, then by an action of $L(F)$ on A we mean a morphism of Lie algebras $\varphi\colon L(F) \to \mathrm{Der}_k(A)$ such that $\varphi(d^p)=\varphi(d)^p$, provided $\mathrm{char}(k)=p>0$. An action of the formal group F on A is a morphism of k-algebras $D\colon A \to A[\![X]\!]$ such that $D(a) \equiv a \bmod (X)$ for $a \in A$, and $F_A \circ D = D_Y \circ D$, where $F_A\colon A[\![X]\!] \to A[\![X, Y]\!]$, $D_Y\colon A[\![X]\!] \to A[\![X, Y]\!]$ are morphisms of k-algebras given by $F_A(g(X))=g(F)$, $D_Y(\sum_\alpha a_\alpha X^\alpha)=\sum_\alpha D(a_\alpha)Y^\alpha$, for a motivation of this notion, see [15]. Let $D\colon A \to A[\![X]\!]$ be such an action. Then, similarly as in the case of an algebraic group action, one proves that the map $\varphi_D\colon L(F) \to \mathrm{Der}_k(A)$ with $\varphi_D(d)(a)=\sum_\alpha a_\alpha d(X^\alpha)|_{X=0}$ for $d \in L(F)$, $a \in A$, and $D(a)=\sum_\alpha a_\alpha X^\alpha$, is an action of $L(F)$ on A.

DEFINITION. An action $\varphi\colon L(F) \to \mathrm{Der}_k(A)$ of the Lie algebra $L(F)$ on a k-alegbra A is said to be F-integrable if there exists an action $D\colon A \to A[\![X]\!]$ of the formal group F on A such that $\varphi=\varphi_D$.

Observe that if $n=1$, $F_a=X+Y$, and $F_m=X+Y+XY$, then an action of $L(F_a)$ (resp. $L(F_m)$) on a k-algebra A is nothing else than a k-derivation $d\colon A \to A$ with $d^p=0$ (resp. $d^p=d$) whenever $\mathrm{char}(k)=p>0$. Moreover, one readily checks that such d is F_a-integrable (resp. F_m-integrable) if there exists a differentiation (= higher derivation) $D=\{D_i\colon A \to A,\ i=0, 1, \cdots\}$ such that $D_1=d$ and $D_i \circ D_j=(i, j)D_{i+j}$ (resp.

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